MIDDLE SCHOOL MATHEMATICS TEACHERS' PERCEPTIONS OF THE STANDARDS FOR MATHEMATICAL PRACTICE EMBEDDED IN CURRICULAR RESOURCES

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This study examines the perceptions of the Standards for Mathematical Practice (SMPs) held by 34 middle school mathematics teachers (MSMTs) as evidenced by their interactions with seven lessons drawn from thinking device (TD) and delivery mechanism (DM) curriculum types. MSMTs' perceptions of the SMPs consistent with their wording in the Common Core State Standards for Mathematics (CCSSM) included a flexible definition of precision beyond calculation. However, MSMTs also possessed a number of perceptions of the SMPs that were at odds with the wording of these standards in the CCSSM. For instance, they considered a curriculum resource's imperative for students to use a tool to be an indication of SMP 5. MSMTs whose district-adopted curriculum was categorized as TD had significantly less invalid SMP justifications than teachers using DM curricula t(34) = 2.41, p = .022.

Keywords: Curriculum, Standards, Middle School Education, Instructional Activities and Practices

The majority of students in the US are situated within educational systems linked to the Common Core State Standards for Mathematics (CCSSM). As we enter the seventh year of CCSSM implementation we find ourselves at an educational crossroads. These crossroads consist of reflecting on what we have learned thus far with regard to CCSSM implementation and determining where we need to go from here to realize these ambitious standards. CCSSM research has investigated the standards themselves (Schmidt & Houang, 2012), examined elementary level mathematics textbooks' alignment to the CCSSM, ascertained teachers' perceptions of the CCSSM (Davis et al., 2014), and described ways that teachers make the Standards for Mathematical Practice (SMPs) explicit to students during classroom instruction (Selling, 2016).

Opfer, Kaufman, and Thomas (2016) investigated the perceptions of a nationally representative sample of K-12 public school teachers in the US and found that teachers reported spending less time on SMP 7 (structure) or SMP 3 (constructing arguments) than other SMPs. In contrast, other research (Davis et al., under review) suggests that teachers consider the SMPs to be components of each lesson that they construct. Opfer and colleagues also found that teachers at the elementary level were more likely to misunderstand SMP 4 (modeling) than secondary teachers. Heck and colleagues (2011) noted that a group of mathematics educators and policy researchers they surveyed were concerned about the separation of the SMPs from content in the CCSSM as well as a lack of clear descriptions of what a trajectory in learning SMPs might look like across grades. An important component that is missing from this research involving teachers' perceptions of the CCSSM is the mediating presence of their district-adopted curricular resources. Mathematics teachers frequently use textbooks (Banilower et al., 2013) and other research that we have completed as part of our larger study suggests that teachers interpret textbook materials vis-à-vis the CCSSM (Roth McDuffie et al., 2017). Given this setting, we were especially interested in the perceptions of teachers using different types of curricula as textbooks that have been referred to as standards-based were created from

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documents that contained processes similar to the SMP. Hence, this study was designed to answer two research questions.

1. What perceptions do a group of middle school mathematics teachers (MSMTs) hold with regard to the SMPs as revealed through their work with two types of curricular resources?
2. How do the SMP perceptions of a group of MSMTs differ by district-adopted curriculum type?

Frameworks

Teachers' Interactions with Curricular Resources

We take the perspective that teachers' curriculum use involves what Remillard (2005) describes as participation with the textbook. That is, we consider teachers to be active interpreters of their curricular resources. Moreover, we consider these interpretations to be governed by teachers' personal resources (e.g., beliefs), the contexts in which they work, orientation, professional identity, students, and curriculum (Stein, Remillard, & Smith, 2007). Additionally, teachers themselves are transformed by their work with curricular resources (Remillard). We situate teachers' work with their curricular resources within Stein and colleagues temporal phases of curriculum use. In particular we use their terminology *intended curriculum* to denote the lesson plans that teachers create from their curricular resources or *written curriculum*. We consider curricular resources to encompass all of the materials associated with a program (e.g., assessment resources) in print or digital forms.

Types of Curriculum Programs

In earlier work (Choppin, McDuffie, Drake, & Davis, 2015) we conceptualized curricular resources based upon monologic and dialogic communication functions. Curricula were categorized as following a *delivery mechanism* (DM) if they serve a monologic function where content is viewed from the perspective of an expert and delivered to novices. Curricula were categorized as *thinking device* (TD) if they serve a dialogic function where the goal involves soliciting the thinking of novices and using this knowledge to move novices towards more complex thinking levels. We use the terminology *TD teachers* and *DM teachers* to denote teachers whose districts have adopted TD curricula types and DM curricula types, respectively, and we place the curriculum type after the teacher's pseudonym in the results section.

Methods

This study is a component of a larger study examining how MSMTs interact with their curricular resources in the context of the CCSSM. This component of the larger study used staged lesson plans (SLPs) to reveal MSMTs' interactions with their curricular resources vis-à-vis the CCSSM. In an SLP, teachers who have used one type of curricular resource for at least one year were given one week to create an intended curriculum from a different type of curricular resource. The SLP was designed to reveal how MSMTs used their district-adopted curricular resources by asking them to plan from a different type of curriculum resource. During the SLPs MSMTs were asked a series of questions involving the CCSSM content standards and SMPs, the SLP curricular resources as well as the district-adopted curricular resources, and the intended curriculum. Participants were purposefully drawn from school districts that had adopted both TD and DM curricula in print and digital forms from both rural and urban middle school settings. A total of 62 different MSMTs working in four different CCSSM states completed 75 SLPs in two waves during the 2013-2014 and 2014-2015 school years. In the initial wave of SLPs, MSMTs were asked to create an intended curriculum from a SLP curriculum resource that was of a different type than the district-adopted curricular resource. In the second wave of SLPs, a selection of MSMTs was asked to create an intended curriculum from

three different lessons from a TD curriculum (described in more detail below). All teachers were asked which SMPs were addressed in the respective intended curricula that they created from these curricular resources. However, only 34 of these teachers were asked specifically to point out in their intended or written curricula where SMPs occurred. The 37 SLPs completed by these teachers form the data set at the center of this study. A total of 21 and 13 MSMTs had been using a DM and TD curriculum for at least one year, respectively.

SLP Curricular Resources

The curricular resources used in the SLPs were drawn from three different curriculum resources, two of which were considered to be TD, and one DM. The two categorized as TD were *Connected Mathematics 3* (CM) (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2014) and *College Preparatory Mathematics* (CPM) (Kysh, Dietiker, Sallee, Hamada, & Hoey, 2013). The DM curriculum was *Glencoe Mathematics* (Glencoe) (Carter, Cuevas, Day, & Malloy, 2013). MSMTs who taught grades 6 were provided with a lesson involving proportional reasoning: (Glencoe – P) (Carter et al., 2013, pgs. 14-27); (CM – P) (Lappan et al., 2014, pgs. 18-23); *Core Connections: Course 1* (CPM1) (Kysh, Dietiker, Sallee, Hamada, & Hoey, 2013a, pgs. 224-227); or *Connections Course 2* (CPM2) (Kysh, Dietiker, Sallee, Hamada, & Hoey, 2013b, pgs. 743-745). MSMT who taught grade 8 were provided with a lesson involving linear functions (Glencoe – LF) (Carter et al., 2013, pgs. 267-277); (CM – LF) (Lappan et al., 2014, pgs. 5-11); or *Core Connections Course 3* (CPM3) (Kysh, Dietiker, Sallee, Hamada, & Hoey, 2013c, pgs. 308-311). We chose roughly equal numbers of grade 7 teachers to work with proportional reasoning and linear functions lessons. For each SLP curricular resource, MSMTs were provided with the student textbook, lesson planning resources, unit planning resources, assessment resources, and a copy of the CCSSM. The number of MSMTs working with each SLP curriculum resource by district-adopted curriculum type appears in Table 1.

Table 1: Participants by Curriculum Type and SLP Curriculum Resource

	District-Adopted Curriculum Resource	
SLP Curriculum Resource	DM	TD
Glencoe – P	–	3
Glencoe – LF	–	5
CM – P	9	–
CM – LF	11	–
CPM1	2	–
CPM2	–	4
CPM3	1	2

Analysis

The data in this study were analyzed through iterative cycles (Miles, Huberman, & Saldaña, 2014). In an initial cycle of analysis we coded data with a variety of broad codes connected to a larger project. In this study, we focused on data coded as 01-MP (mathematical practices), 0-Curric (descriptions of intended curricula or written curricula), and 04-Adapting (adaptations made to the written curricula). Data coded as 01-MP were subsequently coded for each of the SMPs based upon a word or words associated with that SMP. For example, the word *persevere* led us to categorize these data as SMP 1. Interview excerpts including language that was ambiguous (e.g., explore) or potentially could pertain to more than one of the SMPs (e.g., explain) was excluded from analysis. Next, a combination of in-vivo and descriptive coding was used on data associated with each SMP. Last, we examined the codes within each SMP for themes shedding light on MSMTs' perceptions. We determined the validity of each MSMT's justification for the presence of a SMP in the curricular

resources or intended curriculum by comparing the teacher's justification to the written description for each of the SMPs in the CCSSM using techniques similar to Opfer and colleagues (2016) as well as our own previous work (Davis et al., under review) in excerpts coded as 0-Curric. Data coded as 04-Adapting were used to better understand the adaptations made with respect to the SMPs. Our analyses of the written description of the SMPs in the CCSSM led to the identification of SMP 1 and SMP 3 in both the CM – P and CM – LF SLP materials and we calculated the percentage of DM teachers who identified these SMPs in these materials. As the number of invalid SMP justifications made by the 34 teachers in our study met the assumptions of an independent samples t-test we used a two-tailed test to examine the significance of the differences in invalid justifications between TD and DM teachers with an alpha level of .05. Additionally, we calculated the percentage of valid SMPs out of the total SMPs noted for DM and TD teachers.

Results

SMP Perceptions of MSMTs Regardless of Curriculum Type

A common theme running through the majority of MSMTs' responses in the first SMP was multiple approaches. MSMTs perceived that both perseverance and sense making required multiple approaches. Additionally, the majority of teachers noted that problems that required students to make sense of them and preserve in solving them were complex in some way. As Davidson (TD) put it in her SLP, "If they're going to persevere in solving something, it better be something that is going to challenge their thinking in some way."

Only six teachers (all DM) mentioned that their SLP curricular resources contained components of SMP 2. A common theme among responses with regard to this SMP was real-world contexts. That is, some teachers stated that for this SMP to be present students needed to consider the mathematics embedded in a real-world situation. Contextualization and decontextualization were both mentioned by MSMTs in reference to SMP 2.

For many MSMTs, SMP 3 embodied the development of arguments and the careful examination of the arguments produced by others. Instead of arguments, however, MSMTs often stated that students would "discuss," "share," or come to a "consensus" about different ways to solve a problem such as determining which mixture was "most orangey" in the CM – P lesson.

Three of the SLP curricular resources (Glencoe – LF, CM – LF, and CPM3) specifically directed students to create tables, graphs, and equations for a variety of real-world contexts involving linear functions. An example of one set of questions from CM – LF appears below.

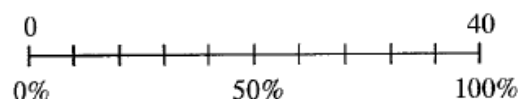| Name	Walking Rate
Alana	1 meter per second
Gilberto	2 meters per second
Leanne	2.5 meters per second

A. 1. Make a table showing the distance walked by each student for the first ten seconds. How does the walking rate affect the data?
2. Graph the time and distance on the same coordinate axes. Use a different color for each student's data. How does the walking rate affect the graph?
3. Write an equation that gives the relationship between the time t and the distance d walked for each student. How is the walking rate represented in the equation? (Lappan et al., 2014, pgs. 6-7).

MSMTs considered questions in the textbook lessons such as the one above to be instances of SMP 4. Christiansen (DM) justified the presence of mathematical modeling in the set of questions above in the following way: "They're making a table as a model. They're making a graph to model the situation. They're also going to be asked to write an equation for each situation. That would be a type of model."

None of the MSMT discussed other aspects of modeling such as moving from the mathematical model back to the real-world context or the assumptions that needed to be made in creating the mathematical model for some real-world situation. For Dietrich (DM) and other MSMTs the presence of what they deemed to be a real-world situation was a necessary and sufficient condition for SMP 4 to occur in the lesson.

Consider the problem appearing in the CPM1 (p. 226) curricular resource.

With your team, you will use the percent ruler shown at right to examine a sample of 40 raisins and peanuts.

Copy the percent ruler onto your paper.
Then use it to determine how many raisins would make 50% of the sample. How many raisins are in 10% of the sample? (Kysh et al., 2013a, p. 226).

MSMTs in our study felt that imperatives asking students to use such tools in the curricular resources was evidence that students were gaining proficiency with SMP 5 as seen in Tyler's (DM) statement: "You have to use appropriate tools strategically in this one because you're working with a percent ruler." Other teachers pointed to the use of tools such as graphing calculators to check their work in creating the graph by hand. Thus, for these teachers they considered such tools as being used strategically by students. Other teachers considered the open-ended use of tools to be a prerequisite for the presence of SMP 5 in curricular resources.

Despite previous research suggesting that many teachers feel that SMP 6 appears in each mathematics lesson they prepare (Davis et al., under review) only 15 out of 34 MSMTs listed SMP 6 as appearing in the SLP curricular resources they were given or their intended curricula. Altogether MSMTs' mentioned precision with regard to communication, accuracy, measurements, labels, reading/creating graphs, calculations, and gathering data.

Several MSMTs described the presence of SMP 7 in their SLP curricular resources. For instance, Dietrich (DM) wanted to draw students' attentions to the structure of percent being compared to 100 in the CPM1 lesson. Similar to the CCSSM architects, MSMTs connected structure with pattern identification (SMP 8). This is perhaps best seen in Christiansen's (DM) examination of the CM – LF curricular resource where she perceived the curricular resources as providing opportunities for students to identify the structure of constant slope within a linear function as embodied in tables, graphs, and equations.

SMP 8 expects students to look for and express regularity in repeated reasoning. MSMTs noted that students would have opportunities to identify the y-intercept and slope of linear functions appearing in tables, graphs, and equations either in the written or intended curriculum.

SMPs Perceptions by MSMTs' District-Adopted Curriculum
We found that 63% of SMP justifications made by teachers using a DM curriculum type and 85% of SMP justifications made by teachers using a TD curriculum type were valid. There was a statistically significant difference in the number of invalid SMP justifications for TD and DM

curriculum types $t(34) = 2.41$, $p = .022$. Regardless of curriculum type MSMTs struggled with correctly justifying SMP 4 and SMP 5. For TD teachers these were the only two SMPs in which they had incorrect justifications. In SMP 1, DM teachers only drew attention to making sense of problems and not to perseverance. Several DM teachers confused complexity with abstraction in SMP 2. For instance, one of the lessons (CM – P) asked students to determine which of four different orange juice mixtures was most orangey and least orangey. Martin (DM) stated that this problem involved SMP 2 and justified the practice in this way, "For number two, with reasoning abstractly and quantitatively, now we're getting into, are they going to think outside the box on certain things as far as 'How am I going to get to what's most orangey or least orangey?'" Interestingly, none of the TD teachers identified SMP 2 in the SLP curricular resources. In SMP 3, DM teachers described checking answers for correctness as an engagement in the development of an argument. Both TD and DM teachers incorrectly justified SMP 4. TD teachers focused on the presence of multiple representations of a function (e.g., graph), but did not connect these to real-world contexts. DM teachers' incorrect justifications in SMP 4 involve the presence of multiple representations or a real-world context. DM and TD teachers both incorrectly asserted that the presence of tools such as a table was evidence of SMP 5. In SMP 6, one DM teacher (Shaw) stated that when students were learning a new method for solving a problem, students did not need to be precise in their work. In SMP 7 and 8 there was not sufficient detail to determine the validity of the justifications of the same two DM teachers (Cartwright and Tyler). Additionally, only 45% of the DM teachers identified SMP 1 and only 35% of DM teachers identified SMP 3 across the CM – P and CM – LF lessons.

A total of eight TD MSMTs were engaged in an SLP for the Glencoe – P or Glencoe – LF lessons. In all of these cases, the teachers saw no indication of the SMPs in the lessons. Granville (TD) summed up her evaluation of the Glencoe – P materials in the following way, "I mean what they say is that the aspects of mathematical thinking, practices 1, 3, and 4 are emphasized in every lesson. I just had a really hard time imagining the way this seemed to play out that they were doing any kind of engaging in any of the practices." Consequently, all eight of the teachers made significant adaptations to their DM curricular resources. The eight TD teachers began their intended curricula by taking problems from the DM curricular resource that were presented as being solved in one way (e.g., table) and providing them to students without an expected solution method. They felt that without presenting a particular method, these problems better embodied SMP 1 as they were less leading, more investigative and would be complex for students to solve. These TD teachers also provided students with a variety of different tools for students to use to solve these problems, which they felt was better connected to SMP 5 due to the fact that students had to choose which tool they would use to solve the problem. Another common theme in the intended curricula among these teachers was the use of cooperative groups whereby students would be expected to solve the problems together and engage in argumentation as they justified their solution methods, thereby engaging in SMP 3.

Discussion and Implications

This study examined a group of MSMTs' perceptions of the eight SMPs as well as how those perceptions differ by MSMTs' district-adopted curriculum type. On the one hand, our findings suggest that the MSMTs we sampled are able to correctly identify SMPs 1, 2, 3, 6, 7, and 8 in a variety of curricular resources or their intended curricula. On the other hand, MSMTs' perceptions about SMPs 4 and 5 were problematic. Teachers using both DM and TD curriculum types did not identify the connection between a real-world context and the mathematical representation or the need to translate from the mathematical representation back to the real-world context. They also did not mention other aspects of modeling such as determining what aspects of the real-world situation should be included in the model and which should be discarded. The MSMTs we interviewed tended

to focus on just one action associated with modeling. In SMP 5, MSMTs considered the mere presence of a tool such as a percent ruler to be sufficient for students to gain proficiency with this practice. That is, they were concerned less about what tools were appropriate for a given situation and what it meant to use those tools strategically.

We assert that MSMTs' difficulties with SMP 4 and 5 are connected to the issue of learning trajectories involving these practices. It is not only important to determine what a particular SMP looks like at a particular grade level (Heck et al., 2011), it is important to understand what types of knowledge and skills comprise an SMP, when those should best be taught, how those skills are sequenced, and what curricular resources embodying these activities look like. For example, we would expect that an initial step in developing competency in SMP 5 would involve understanding how to use particular tools. However, as several MSMTs in our study noted in their intended curricula, students need to be asked to complete tasks where they must choose which tool is best suited for the task and justify that use of tools. As we stand at these CCSSM educational crossroads, an important next step in supporting teachers in bringing these standards to life involves providing professional development for teachers in these two SMPs, articulating a set of competencies associated with these skills, and developing curricular resources that embody these skills set within a reasoned trajectory.

We found that the DM teachers in our sample were less successful in identifying SMPs in their curricular resources or their intended curricula than the TD teachers we interviewed. This suggests that the understanding of SMPs exhibited by the DM teachers in our sample is different from the understanding of the SMPs held by TD teachers we interviewed. Consequently, we would expect that DM teachers' classrooms would provide fewer opportunities for students to engage in the SMPs for two reasons. First, DM teachers may experience difficulty in ascertaining when these practices occur in their curricular resources. Second, as the TD teachers demonstrated when they engaged with DM curricular resources, these materials may simply not provide students with many opportunities to engage in the SMPs.

TD teachers engaged in SLPs involving DM curricular resources demonstrated that invigorating these materials with SMPs is not an easy task. In the eight cases where TD teachers were engaged in this work, their intended curricula bore only a slight resemblance to the DM curricular resources from which they were drawn. If educational policy advocates wish to take the development of students' SMP proficiency seriously or as the CCSSM implores, connect content to the SMPs, our study suggests that not all types of curricular resources are created equal. Indeed, bringing the SMPs to life in the classroom may require that we seriously consider the adoption of TD curriculum resources and the concomitant professional development they require as we reflect on our location at the CCSSM crossroads and where we go from here.

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